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Asymmetric localization in disordered Landau bands

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Abstract

We show that, due to band mixing, the eigenstate localization within the disordered Landau bands gets an asymmetric structure: the degree of localization increases in the lower part of the band and decreases in the upper one. The calculation is performed for a two-dimensional lattice with the Anderson disorder potential and we prove that this effect is related to the upper shift of the extended states within the band and is enhanced by the disorder strength. The asymmetric localization and the energy shift disappear when the interband coupling is switched off.

(Some figures in this article are in colour only in the electronic version)

The localization effect in Landau bands has attracted much interest since the discovery of the quantum Hall effect (QHE) [1–9]. When an impurity potential is present, the initially degenerate Landau levels of the two-dimensional (2D) system turn into broad bands. In contrast to the zero magnetic field case when there are no extended states in two dimensions [10], extended states are present at the centre of each Landau energy band [1]. The early studies performed for the continuous Hamiltonian model with neglected inter-Landau band mixing [2–4] show that the generic picture of the 2D Landau bands contains localized states in the band tails and extended states in the middle. However, in a real 2D system the band separation may become smaller than the band width, so the above approximation is not always valid. It has been shown that the mixing between different Landau bands, that generally comes from the presence of disorder, is related to the energy shift of the extended states from the central position of the band [11–17] or it may have a delocalization effect when the states with opposite chirality are coupled [18].

The aim of the paper is to describe the influence of band mixing on the localization properties in the Landau bands. The lattice model captures this relevant feature, and this happens because the discrete Landau model (initially solved by Hofstadter for the pure case [19]) automatically takes into account the interband coupling. We note the two effects put into evidence in the previous works. One is the energy shift between the position of the extended states and the peak of the density of states that was explicitly calculated in the lattice

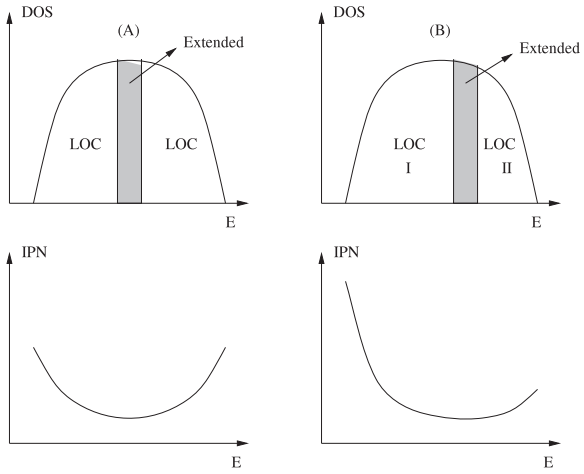

Figure 1. (A) The symmetric Landau band: in the absence of the interband coupling the extended states are positioned in the middle of the band, and the inverse participation number (IPN) is a symmetric function within the band. (B) The asymmetric Landau band: in the presence of the interband coupling the extended states are pushed upwards and the IPN becomes asymmetric, indicating that on the left (LOC I) there are many strongly localized states, while on the right (LOC II) the localized states are less and less localized. The IPN measures the degree of localization.

model by [22], this effect being related to the floating-up conjecture [23, 24]. Furthermore, for the same model, one notes an asymmetric behaviour of the eigenstate localization in Landau bands. This was reported in [20] by the calculation of the localization length within the outer Landau band, and, more recently, in [21] by the calculation of the inverse participation number within the Landau bands. In this paper we make a step forward and show that the quantum origin of this asymmetric behaviour of the eigenstates localization is the mixing of disordered Landau bands. We show this by the projection of the 2D discrete Hamiltonian on the nondisordered Landau levels. It will be also shown that this effect is intimately related to the shift of the extended states. This means that both of the effects disappear when the interband coupling is switched off. The structure of a Landau band with and without band mixing is sketched in figure 1.

For our calculation we use the following 2D Landau Hamiltonian written in the discrete basis of a 2D rectangular lattice:

$$H(\phi) = \sum_{n=1}^{N}\sum_{m=1}^{M}[te^{2\pi i m\phi}|n,m\rangle\langle n+1,m| + t|n,m\rangle\langle n,m+1| + \text{H.c.}] + V, \tag{1}$$

where V is the Anderson disorder potential:

$$V = \sum_{n=1}^{N}\sum_{m=1}^{M}\epsilon_{nm}|n,m\rangle\langle n,m|. \tag{2}$$

The discrete points (n, m) define the 2D rectangular lattice with surface $L^2 = N \times M$ and lattice constant a. $\{|n,m\rangle\}$ with $n = 1\cdots N$ and $m = 1\cdots M$ is the discrete vector basis and it generates the Hilbert space of the one-electron states. Periodical boundary conditions are used, meaning that $|n, M+1\rangle = |n, 1\rangle$ and $|N+1, m\rangle = |1, m\rangle$ (2D toroidal geometry). The perpendicular magnetic field in the Landau gauge $A = (-By, 0, 0)$ is introduced by Peierls substitution in the hopping elements along the $x = na$ direction, $t \to t\exp\{2\pi i m\phi\}$, where ϕ is the magnetic flux through the unit cell a^2 of the lattice measured in quantum flux units

$\phi_0 = h/e$. In (2) the energies ϵ_{nm} represent random variables uniformly distributed in the energy interval $[-W, W]$. W is the amplitude of the Anderson disorder potential (or disorder strength); t is the energy unit and is set to one. For commensurate values of the magnetic flux, the eigenstate spectrum of the pure system ($V = 0$) exhibits the well-known Hofstadter butterfly structure [19]. In the numerical calculation we set the flux value as the ratio $\phi = 1/p$ and the system size as $L^2 = (integer \cdot p)^2$. In this case the eigenstates of the 2D system are grouped in p discrete Landau bands, every band having $n_\mathrm{b} = L^2/p$ degenerated eigenstates. To each energy level ϵ^0_α, with the band index $\alpha = 1 \cdots p$, there correspond n_b degenerate eigenvectors $|\Psi^0_{\alpha i}\rangle = |\alpha i\rangle$ with $i = 1 \cdots n_\mathrm{b}$. The range of validity of the lattice model is extensively discussed in [25], where it is shown that the model correctly describes the 2D physical system in the bottom-left corner of the spectrum. In the presence of the disorder potential the degenerate energy level ϵ^0_α turns into the broad energy band $\{\epsilon_{\alpha i}\}$ with $i = 1 \cdots n_\mathrm{b}$. We study the degree of localization of the nondegenerate eigenstates $|\Psi_{\alpha i}\rangle$ by calculation of the inverse participation number, IPN, which is defined as

$$\mathrm{IPN} = P_{\alpha i} = \sum_{n,m} |\langle n, m|\psi_{\alpha i}\rangle|^4. \tag{3}$$

$P_{\alpha i}$ varies from $1/L^2$ for the extended states, when the electron wavefunction spreads out over the whole surface of the plaquette, to 1 for the strong localized states. The nature of the eigenstates can also be checked by calculation of the variance of the level spacing distribution, δt [26].

In this work we put into evidence the role of the interband interaction. To this end, we write the Hamiltonian (1) in the vector basis of the 2D pure system $\{|\alpha i\rangle\}$:

$$H(\phi) = \sum_{\alpha=1}^{p}\sum_{i=1}^{n_\mathrm{b}} \epsilon^0_\alpha |\alpha i\rangle\langle\alpha i| + \sum_{\alpha=1}^{p}\sum_{i,j=1}^{n_\mathrm{b}} V_{\alpha i,\alpha j}|\alpha i\rangle\langle\alpha j| + c\sum_{\alpha\neq\beta=1}^{p}\sum_{i,j=1}^{n_\mathrm{b}} V_{\alpha i,\beta j}|\alpha i\rangle\langle\beta j|. \tag{4}$$

In this representation the disorder potential V becomes a sum of two terms, corresponding to the intraband and interband coupling (the second and the third term in (4), respectively). $V_{\alpha i,\beta j}$ are the matrix elements of the Anderson potential V written in the basis of the eigenfunctions of the ordered system $\{|\alpha i\rangle\}$. They are random variables as well, and their values are proportional to the disorder strength W. The coupling constant c is introduced for convenience.

We discuss first the situation when the parameter c is set to zero in (4), meaning that only the intraband coupling is taken into account. A picture of the disordered bands for this case is given in figure 2. The density of states (DOS) for the first two bands is shown in figure 2(a), and its profile has a semielliptic shape. Let E_b be the energy where the DOS reaches its maximum, which in this case is located in the middle of the band. The level spacing distribution is calculated by averaging over different disorder configurations in the manner described in [26]. It is known that the extended states belong to the unitary Wigner–Dyson ensemble $\beta = 2$ with the variance of the level spacing $\delta t = 0.42$, while the localized ones are distributed according to the Poisson law with the variance $\delta t = 1$ [27]. The calculation of the level spacing variance in figure 2(b) shows the presence of extended eigenstates in the middle of the disordered bands, for energies E around E_b. For energies towards the band edges the states become localized and there is a continuous crossover from a unitary Wigner–Dyson distribution to a Poisson distribution as in [26]. Obviously, the most localized states are at the edges of the band, where δt increases to higher values. We note that in the thermodynamic limit the extended states in the band centre collapse into a single energy level [9].

We complete the picture of the eigenstates localization for uncoupled bands ($c = 0$) showing in figure 2(c) the values of the IPN. By varying the energy from the central position

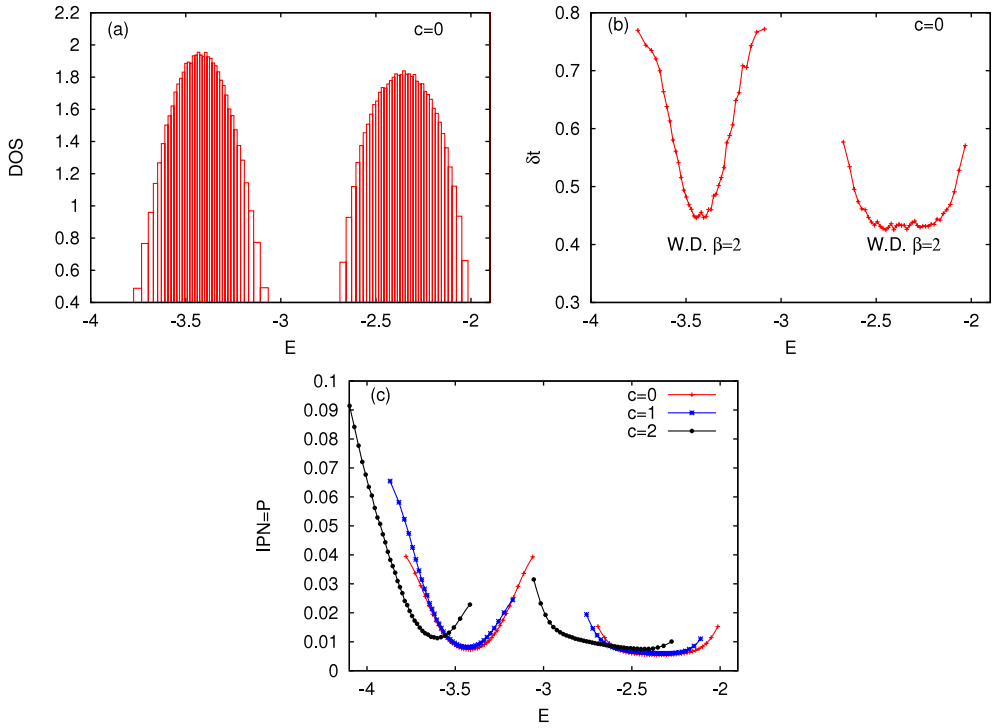

Figure 2. The density of states (a), the level spacing variance δt (b), and the inverse participation number (c) for the first two Landau bands versus energy. ($L^2 = 20^2$, $W = 2$, $\phi = 0.1$.) The DOS and δt are presented only for the symmetric case with interband coupling constant $c = 0$. IPN is presented for $c = 0$, 1 and 2. In (b), when $\delta t = \sqrt{3\pi/8 - 1} \simeq 0.42$, the eigenstates correspond to the extended states of the unitary Wigner–Dyson ensemble ($\beta = 2$). (The configuration average is performed over 5000 samples.)

of every band, the IPN increases, indicating the increased localization of the eigenstates. Let E_c be the energy with the lowest IPN value in the middle of the band. This is the energy of the most extended state, where, in the thermodynamic limit, the localization–delocalization transition takes place. For the case discussed here, when $c = 0$, the critical energy corresponds to the maximum of the DOS ($E_c = E_b$), and the IPN is symmetric within the band. This is what we call the symmetric case. These properties are not preserved any longer when $c \neq 0$, as we can already notice in figure 2(c) (see the IPN curves for $c = 1$ and 2).

In what follows, we are interested in finding out how the localization properties evolve with the interband coupling c. In figure 3 we show the result of the numerical calculation for the inverse participation number and density of states as function of $E - E_c$ for three values of the interband coupling constant, $c = 0$, 1 and 2. For the first band, the IPN curves are depicted in figure 3(a). By the definition of E_c, the IPN takes the minimum value at $E - E_c = 0$. One remarks that the symmetry of the IPN is lost for nonvanishing coupling constant $c = 1$ and 2, i.e. in the case of band mixing. Compared to the uncoupled case ($c = 0$), the increased values of IPN for $E < E_c$ at $c \neq 0$ indicate an increased degree of localization in the lower part of the band. The opposite is true in the upper part, where the states become less localized.

In figure 3(b) we depict the DOS of the first Landau band. For $c = 1$, the band is shifted downwards, meaning that the maximum of the DOS does not correspond to E_c, but it occurs at

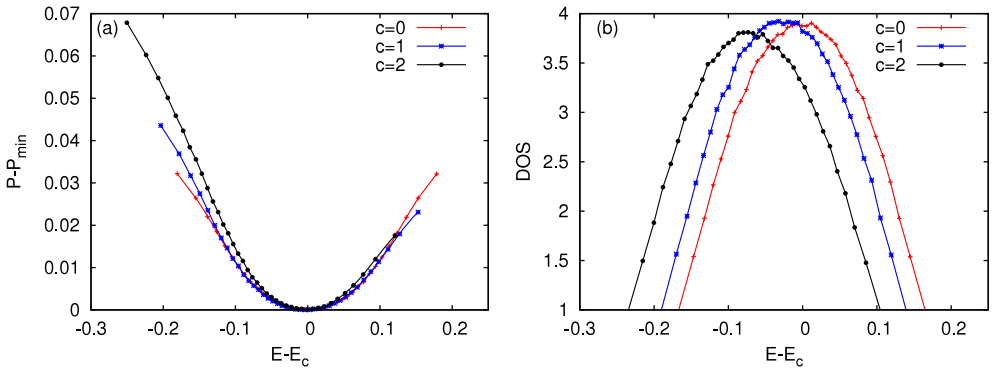

Figure 3. (a) The inverse participation number $P - P_{\min}$ and (b) the density of states for the first Landau band plotted versus energy at different values of the interband coupling constant: $c = 0$ (crosses), $c = 1$ (stars) and $c = 2$ (dots). ($L^2 = 20^2$, $W = 1$, $\phi = 0.1$.) $P_{\min}$ is the minimum value of the inverse participation number, while E_c is the energy where this value is reached, i.e. $P_{\min} = P(E_c)$.

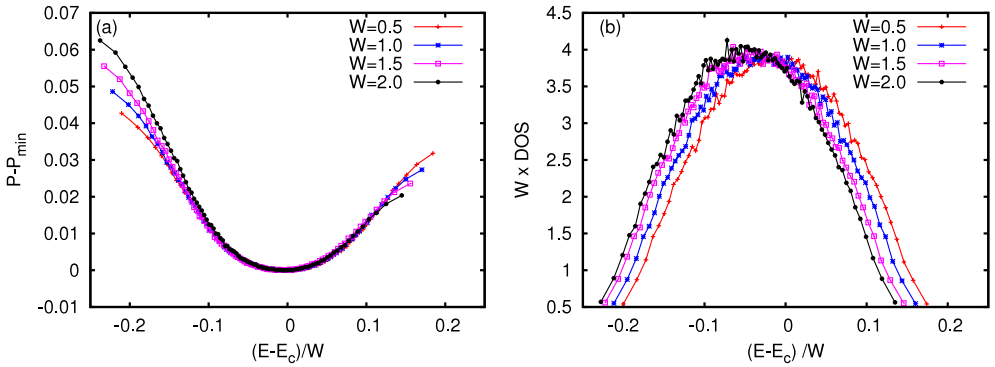

Figure 4. (a) The inverse participation number $P - P_{\min}$, and (b) the density of states (DOS) for the first Landau band versus energy at different disorder strength $W = 0.5$ (crosses), $W = 1$ (stars), $W = 1.5$ (squares), and $W = 2.0$ (dots) ($L^2 = 30^2, \phi = 0.1, c = 1.$) $P_{\min}$ is the minimum value of the inverse participation number, while E_c is the energy where this value is reached, i.e. $P_{\min} = P(E_c)$. Note that the energies are scaled by the disorder strength W.

a lower energy $E_b < E_c$. The shift increases for $c = 2$. This means that the critical energy is moving up in the band when the interband coupling is present. This asymmetric behaviour is preserved for all the bands contained in the lower half of the spectrum.

Once we have established the relation between the interband coupling and the asymmetry of the Landau bands, we are interested now in learning how this property depends on disorder. We keep $c = 1$ fixed in (4) and increase the amplitude of disorder W. Since the interband coupling in the discrete Hamiltonian is due to the presence of the disorder, we expect the shift of the critical energy to also be dependent on the disorder amplitude.

Figure 4 gives the inverse participation number and the density of states as a function of $(E - E_c)/W$ for different values of W. One notices in figure 4(a) that, for any disorder, in the domain of extended states around E_c the inverse participation number $P(E)$ can be expressed as $P(E) - P(E_c) = f((E - E_c)/W)$. Deviations from this law are noticed at the band edges.

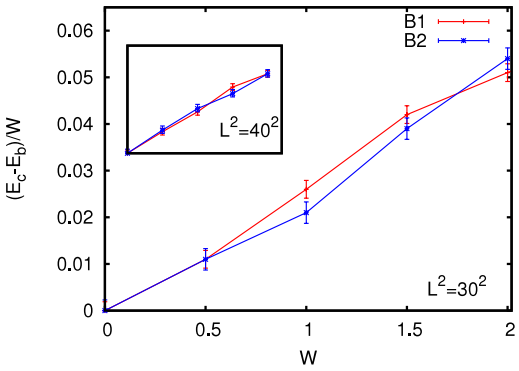

Figure 5. The shift of the critical energy E_c versus disorder strength W. ($L^2 = 30^2$, $\phi = 0.1$, $c = 1$.) The results are plotted for the first band (B1) and for the second band (B2). The numerical results obtained for $L^2 = 40^2$ are presented in the inset.

By increasing the disorder amplitude W, the lower-energy states of the band become more localized, but the higher-energy states of the band become less localized. At the same time, the extended states move towards the upper edge of the band, so the critical energy E_c does not correspond any longer to the maximum of the density of states. This can be seen in figure 4(b), where one notices the band shift with increasing disorder strength. The critical energy shift $E_c - E_b$ as a function of disorder is shown in figure 5 for the first two bands in the spectrum. For the present model, $(E_c - E_b)/W$ exhibits a linear dependence on the disorder strength W; this result confirms the previous numerical works, where the energy shift was shown to be proportional to the square of the band width [17, 22]. For the model used, with Anderson disorder potential, we note no significant difference of the critical energy shift for the two bands depicted. The error bars ΔE_c in the figure 5 correspond to an error of the minimum of the IPN value of 1%. The numerical calculations were repeated for an increased system size, the results being the same.

In conclusion, we have shown that band mixing gives rise to the asymmetry of the localization properties in the Landau bands: the inverse participation number (which measures the degree of localization) becomes an asymmetric function within the band, indicating that the degree of localization increases for the states in the lower-energy part of the band and decreases for the states in the upper part. At the same time the critical energy (the most extended state) in each Landau band does not correspond to the maximum of the density of states but it is shifted to higher energies. These properties are specific to the many-band model and are quite different from the properties of the one-band model which exhibits only symmetrical features. In a large range of energies the inverse participation number $P(E) - P(E_c)$ can be expressed as a universal function of $(E - E_c)/W$.

Finally, we note that the above-discussed asymmetry is the argument used in [28] to explain the displacement of the integer values of the filling factor from the middle point of the Hall plateau in a Si-MOSFET sample.

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